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Custodian:	A. Ealet		

Authors:	**Function:**	**Signature & date :**
A.Chapon	Detector Simulation Responsible	DocuSigned by: A. Chapon 6B6DD786507E4DD... 6/9/2013
A.Tilquin	Simulation	
A.Ealet	NISP spectro Instrument scientist	DocuSigned by: A. Ealet BB64C2218A4A498... 6/9/2013
Approved by:	**Function:**	**Signature & date :**
A.Ealet	NISP spectro Instrument scientist	DocuSigned by: A. Ealet BB64C2218A4A498... 6/9/2013
Authorised by:	**Function:**	**Signature & date :**
T. Maciaszek	NISP project manager	DocuSigned by: thierry maciaszek 610833C2BF2E41D... 6/11/2013
Y. Mellier	Euclid Consortium Lead	DocuSigned by: Y. Mellier 31AAAB8DB0384B4... 6/11/2013

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Draft 1	22/05/13		First draft	
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1 Introduction

1.1 Purpose

This document aims at studying impact of both the control of electronic noises and the control of pathologic behavior of some pixels on the flux measurement of the NISP detectors and define a way to detect anomalies.

Photometric errors will depend of the data acquisition scheme of the NISP detector, which is currently based on particular readout modes: the photometry channel is using a Fowler-16 mode or a multi-16x3 (TBC) with typical exposure time below 100s and the spectroscopic mode is defined to use a multi-4x37 mode in 560s or a multi-16x15 (TBC). These modes are currently implemented via a linear regression on board of the acquisition temporal ramp.

We describe in this note a method based on a weighted least-square fit, which take the error of the groups into account and give then exactly poisson errors in the poissonien regime. We compare the error in the flux estimation in the pixel using this method from the one used in the linear regression as presented in RD2.

We then describe a χ^2 quality test based on this least-square fit. Its value is borne out to a well define distribution in most cases and strongly increases in some uncontrollable others. It can then detect pathologic behaviors as cosmic rays in the ramp, non linearity, telegraph noise, reset anomalies.

2 Documents

2.1 Applicable documents

AD	Title / Author	Document Reference	Issue	Date

2.2 Reference documents

RD	Title / Author	Document Reference	Issue	Date
0	NISP Acronyms List	EUCL-IAP-LI-1-001	2.0	04/05/2013
1	Detectors for the JWST near-infrared spectrograph I readout mode, noise model and calibration considerations	Rauscher et Fox, Astronomical society of the Pacific, 119: 768-786		2007
2	NISP FPA On-Board Pre-Processing Description	EUCL-OPD-NPS-TN-00134	1.0	2012

3 Acronyms

See RD0

4 Document Overview

In this document, we present the basic formula used to compute the error on the flux in one pixel on the IR detector using a weighted least square fit. We do a realistic ramp simulation where we simulate on each point the up the ramp / multi accum mode including CDS noise as is expected at present in NIR detectors.

Taking into account this model, we compute the final expected error of the fit and the value of the χ^2 for standard cases and some pathologic cases. We show that the value of the χ^2 on the fit is well predictable if all points of the ramp are consistent with the expected ones, but is strongly larger in most of other cases. That should permit to detect behaviors which induce an error large enough to impact the final error budget, and it must be reliably monitored.

We describe then a method to use this χ^2 as a quality test and propose to send it on ground to detect anomalies. We conclude on recommendations to use this inside the on board processing.

5 Key Formula of the least square fit

As needed for science results, the error on each pixel fluxes has to be correctly estimated. In the case of NIR pixel detector on board, the final flux noise error is not a direct Poissonien value but depends of the temporal fit and of the readout mode.

We take here the assumption of a multi-accum (m,n) read out mode as in spectroscopy. In the case of a full ramp of m frames with n groups, the fit error has to take into account all noise contributions, ie the error of each frame. In particular, these detectors have a high single readout noise by frame which has a non negligible effect on the final noise. Because of the poisson noise, the frames are highly correlated and this has to be taken into account to calculate the fit error.

5.1 Signal and error of co-added frames

We first assume that frames will be co-added. When coadding m frames, the signal s_i with dt_{read} between each frame, is for the frame i:

$$s_i = \frac{1}{m} \sum_j^m s_j(read)$$

and the error is

$$\epsilon_i^2 = s_i + \frac{\sigma_r^2}{m}$$

The impact of the co-adding operation is illustrated by figure 1. In the case of a positive readout-error σ_r, it enables to reduce error bars on groups, to the detriment of the number of points to fit in a ramp. In fact, taking the average signal for co-added frames reduce correlation which is a big advantage to do a weighted least square fit.

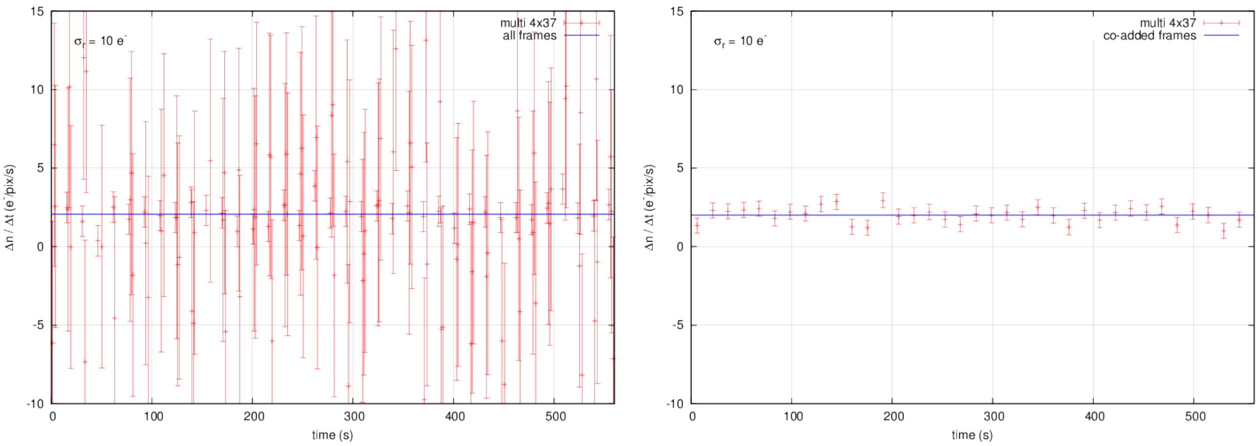

Figure 1: *effect of co-adding on the errors on each groups (multi-4x37 readout mode): left is the difference of groups (s_i-s_{i-1}) without coadding (4x37 points) , right after coadding (37 points)*

To be fully correct, the co-adding should be weighted to correct the fact that the flux is increasing during the ramp. We have seen by checking the flatness of the χ^2 probability of the fit, that this is not impacting strongly the χ^2 analysis or the final result, and we have decided to not use the weighted values in the following sections.

5.2 The chi2 estimator

5.2.1 Without readout error

We first assume a case where we are dominated by signal then the readout error is taken as negligible. In the s_i space, signal is cumulative, thus individual measurement are strongly correlated. We choose to work in Δs space because poisson fluctuations are uncorrelated and we are less sensitive to any normalisation effect. The χ^2 can be written as:

$$\chi^2 = \sum_i \frac{((s_i - s_{i-1}) - f_{fit})^2}{f_{fit}}$$

Notice that the poisson error is taken from the fitted value, because poisson fluctuation is from expected (fitted) signal. Fitted flux value can be calculated using the first derivative of the χ^2:

$$\frac{\partial \chi^2}{\partial f_{fit}} = 0$$

Simple algebra gives for n groups:

$$f_{fit} = \sqrt{\frac{\sum_i (s_i - s_{i-1})^2}{n-1}}$$

The error on flux is obtained with the second derivative of the χ^2:

$$\sigma_f = \sqrt{\frac{f_{fit}}{n-1}}$$

Notice the χ^2 can be computed at any step of the ramp using only two intermediate informations, the square of the flux difference and the first coadded image:

$$\chi^2 = 2((n-1)\sqrt{\frac{\sum_i (s_i - s_{i-1})^2}{n-1}} - (s_n - s_1))$$

5.2.2 With the readout error

Our simulation shows that for a low flux, the fit error is systematically too high of the order of 0.3 e^-/pix/s. This is due to the fact that the readout error introduces correlation between flux difference. To solve that problem we should add the readout error to the poisson error:

$$\sigma_{\Delta s} = \sqrt{\Delta_s + 2\frac{\sigma_r^2}{m}}$$

The new χ^2 expression is:

$$\chi^2 = \sum_i \frac{((s_i - s_{i-1}) - f_{fit})^2}{f_{fit} + 2.\frac{\sigma_r^2}{m}}$$

Let's then define f'_{fit} to be $f_{fit} + 2.\frac{\sigma_r^2}{m}$.

Then, from the previous equation, we find:

$$f'_{fit} = \sqrt{\frac{\sum_i (s_i - s_{i-1} + 2\frac{\sigma_r^2}{m})^2}{n-1}}$$

with the new error on the flux to be:

$$\sigma'_f = \sqrt{\frac{f'_{fit}}{n-1}}$$

The new χ^2 expression is then:

$$\chi^2 = 2 \times \left((n-1)\sqrt{\frac{\sum_i (s_i - s_{i-1} + 2\frac{\sigma_r^2}{m})^2}{n-1}} - (s_n - s_1) - 2(n-1)\frac{\sigma_r^2}{m} \right)$$

Our simulation showed that systematic error is reduced at a level of 0.01 e^-/pix/s whatever is the flux. However the error is always overestimated for very low fluxes (<0.1 e^-/pix/s).

These equations are supposing that read errors σ_r are not correlated. Even if it is negligible, it is an approximation. But to take correlations into account implies to solve in flight a polynomial whose degree is n, the number of groups per exposure. We find the current result is enough accurate for the science measurement.

5.3 Comparison with the current on board implementation

When doing a linear regression as proposed in JWST [RD1], a good estimate of the slope is done and is given by the expression used in [RD2] :

$$\Delta s_{we} = \frac{6}{n(n+1)(n-1)}\left(2\sum_i i s_i - (n+1)\sum_i s_i\right)$$

In this case, no error is used and calculated. To solve this, a noise formula has been derived, often known as the Rauscher formula [RD1]. This formula has been used also in Euclid WE estimation to verify the fit implementation in simulation ([RD2]). It takes into account the correlation between groups and the real noise in each group. This error on the slope a can be written as:

$$\Delta a^2 = \left(\frac{12(n-1)\sigma_{read}^2}{mn(n+1)}\right) + \left(\frac{6(n^2+1)}{5n(n+1)}(n-1)dt_{group}a\right) - \left(\frac{2(m^2-1)}{mn(n+1)}(n-1)dt_{frame}a\right)$$

where σ_{read} is the error on one frame, dt_{frame} the time between 2 frames and dt_{group} between 2 groups. The third term is due to correlation between points of the ramp. The final error will then depend on details of the readout mode as already emphasized in [RD2]. This formula is anyway only an approximation at ~10-20 % level and is not giving a poisson estimation of the final error as it can be seen on figure 2 but a higher value. This can be seen as loosing some flux sensitivity (upper SNR).

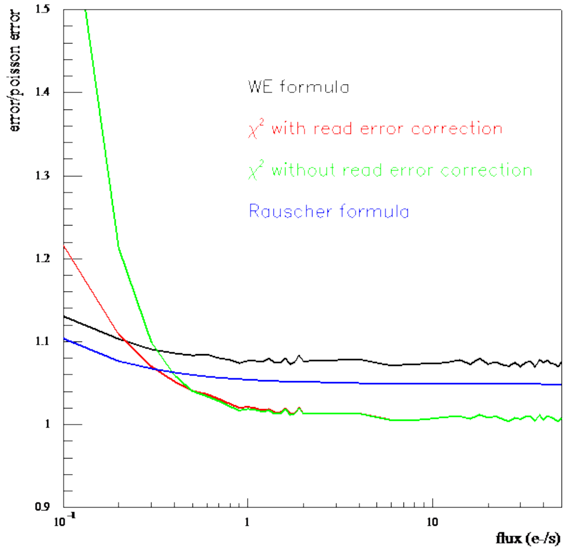

Figure 2 : *the comparison of the fit error with the different methods*

Also, it can be see in figure 2 that the fit error is larger for flux < 0,1 e/s for the method presented here. This is because all correlation of the readout noise are not taken into account.

Let remarks finally, that the fit value s_{WE} itself is correct and can be used in the χ^2 expression in place of f'_{fit}.. Only the error is too high because of the correlation. This means that the χ^2 can be computed whatever the least square method.

6 χ2 as a quality test of the least square fit

The χ^2 is a parameter which have a mean value which is egal to the number of freedom of the fit -1 and a rms which is given by the number of freedom/2. The number of freedom is the number of point of the fit. Then, if the χ^2 deviates from the predicted value, it is a sign of an anomaly and then this can be see as a quality test of the fit. We present here a way to use it to identify presence of anomalies occurring during an exposure.

The χ^2 depends not only on the number of groups but also on the number of co-added frames per group. Higher is the number of frames per group, lower is the error due to readout-error, and better is the rejection power of anomalies, based on the χ^2 value.

Then in this study, we keep a co-added approach of ‘m’ frames in ‘n groups. First, the **rejection efficiency** has been tested in the case of the currently implemented multi-4x37 readout mode and for different fluxes. In this case the predictable value should be center around a mean value of 33 with a RMS of 18. Then, we do an optimisation of the number of groups in the next section.

6.1 Normal χ^2 case

We built the distribution of the χ^2 values in the case of the currently implemented multi-4x37 readout mode and for different fluxes. An example of such fit is shown on figure 3.

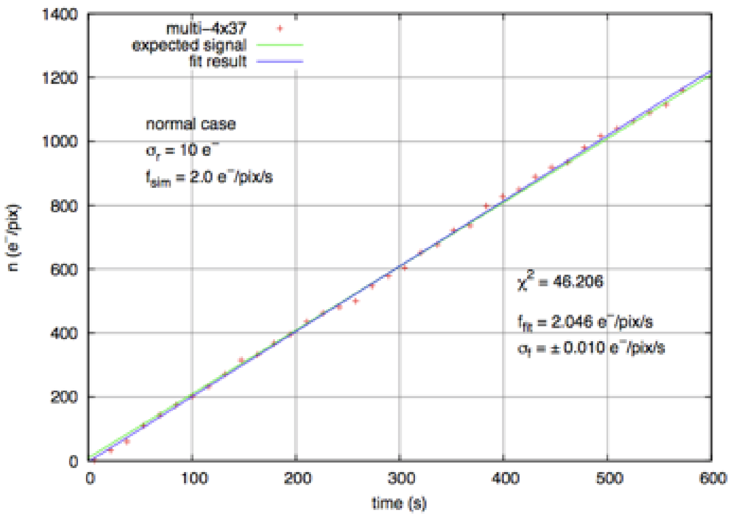

***Figure 3**: Linear fit and associated χ² test quality – normal behavior (expected flux: 2 e⁻/pix/s)*

The χ^2 distribution is shown on Figure 4. The distribution is centered at 33, about 95% of pixels have a χ^2 lower than 44 (3 sigma) and 99% present a χ^2 value lower than 51 (5 sigma) , regardless of the flux. The predicted value of the χ^2 is slightly shifted from 35 to 33 due to the co-adding effect. On the right, we show the probability of the χ^2, which can be used to detect pathologic pixels.

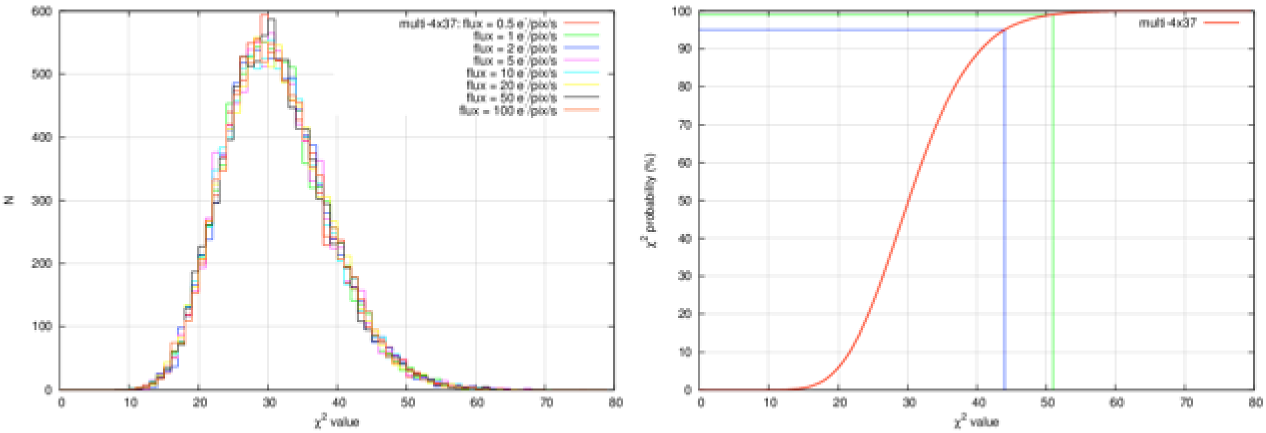

Figure 4*: χ^2 distribution in the case of multi-4x37 readout mode on left and the probability of χ^2 on right.*

6.2 χ^2 in case of anomalies

We present some examples of possible anomalies to show their effect on the χ^2.

The first case is the interaction of a cosmic ray in a pixel with different amplitudes randomly distributed in time, including between frames of a same group. The second is a non-linearity response equal to 20% compared to the linear behavior. The last is a ‘bit switching’ effect.

Impact of cosmic rays.

The interaction of a cosmic ray in a pixel is shown on figure 5. The flux measured in such pixel is higher than the expected one.

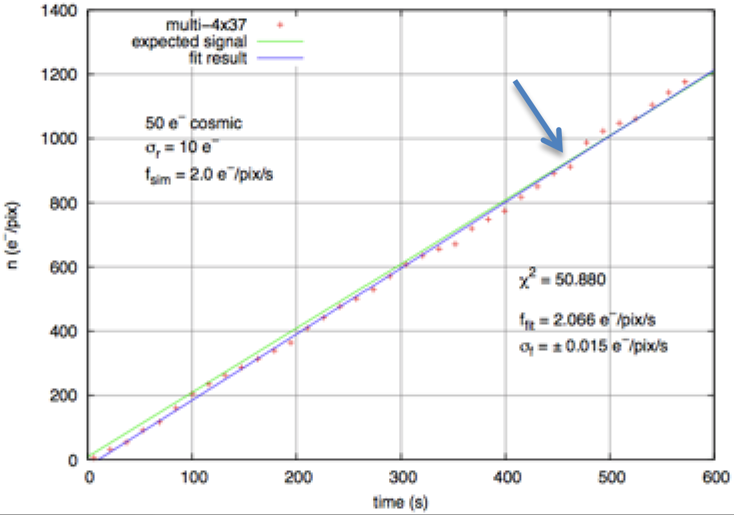

***Figure 5**: Linear fit and associated χ^2 test quality – Cosmic of ~50 e- (indicated with an arrow) (expected flux: 2 e-/pix/s)*

For a given flux, the χ^2 distribution can be plotted considering cosmic rays with different amplitude randomly distributed in time. That is what illustrates figure 6.

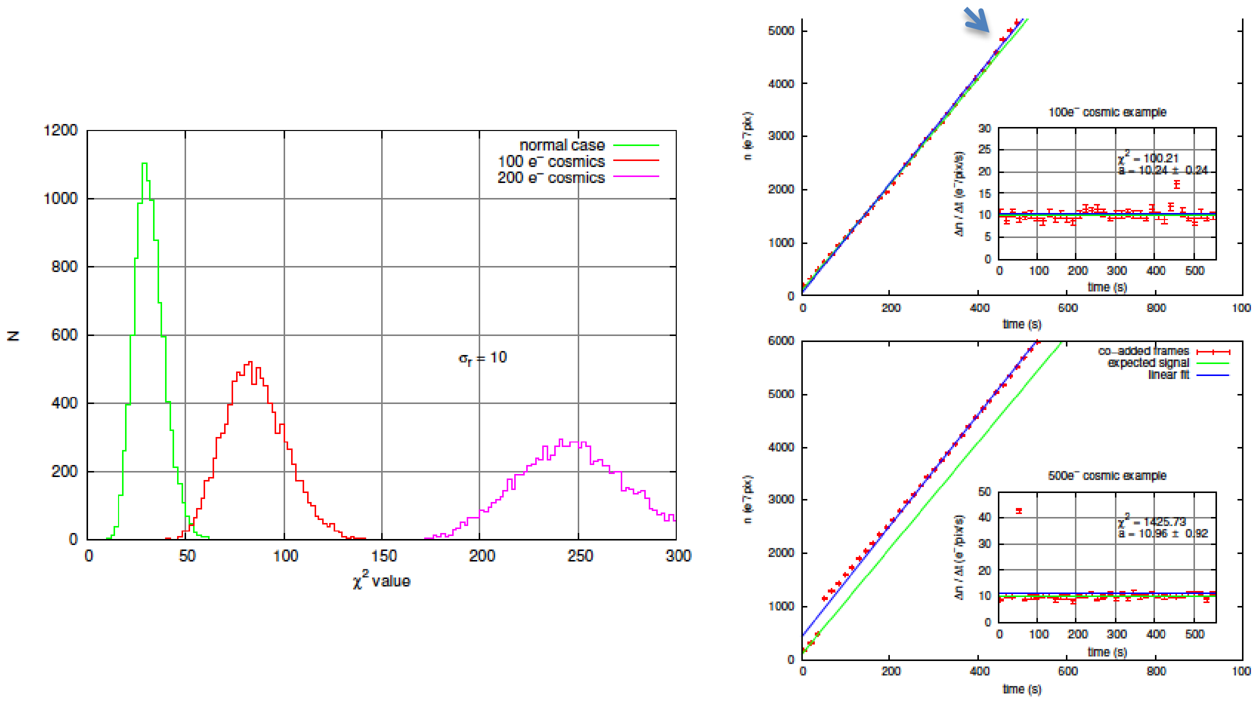

***Figure 6**: χ^2 distribution in the case of multi-4x37 readout mode and a flux of 10 e-/s – Cosmic rays*

We emphasize that the advantage of the weighted least square fit method using difference of groups in the ramp, is that the result is identical if the cosmic falls between groups or inside a group.

Non linear effect

Similarly, the same study is applied to a pixel which present a non-linear response. In such pixel, the measured flux is lower than the expected one due to a decreasing of the signal accumulation at the end of the ramp. That is illustrated on figure 7.

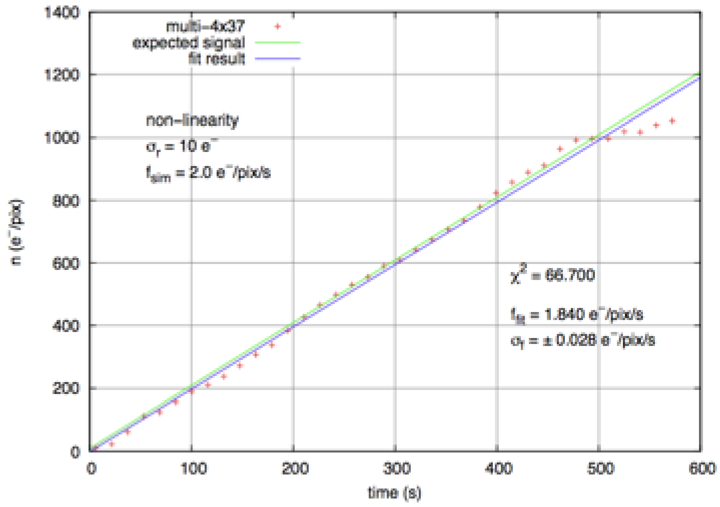

Figure 7*: Linear fit and associated χ^2 test quality – Non-linear response (expected flux: 2 e⁻/pix/s)*

This kind of effect implies a larger value of χ^2. If a pixel has a non-linearity response higher than 20% compared to the linear behavior, it is well seen using the χ^2 test.

Bit switching

A bit switching, which may be caused by cosmic ray in electronic, is simulated by a missing point in the ramp. The case of a missing point is shown on figure 8.

Only some cases are discussed here, but we have simulated enough different cases that can make us comfortable to say that the χ^2 can have the power to implement a rejection criteria that can detect many other kind of anomalies like reset anomaly or other unknown behaviors which can induce a significant systematic error.

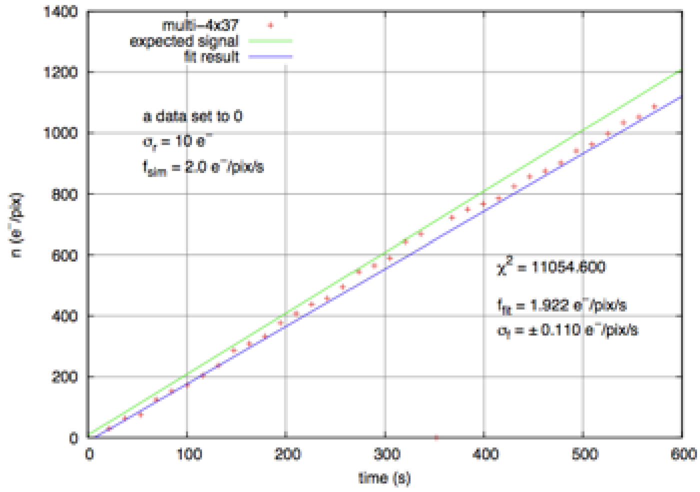

***Figure 8**: χ^2 distribution in the case of multi-4x37 readout mode – one missing point*

6.3 Detection and control of anomalies

We propose here a method to use the χ^2 on ground to define 'wrong fits'. By simulation, we can estimate, for the science range of fluxes, the level of cosmic amplitude that should be detected to not introduce a systematic error. As previously states, the error will depend of the signal flux but, we can define one single value of the χ^2, above which, the pixel is flagged and this, whatever the flux. This is illustrated on Figure 9, using the standard readout mode 4x37 for which the mean value of the χ^2 is 33.

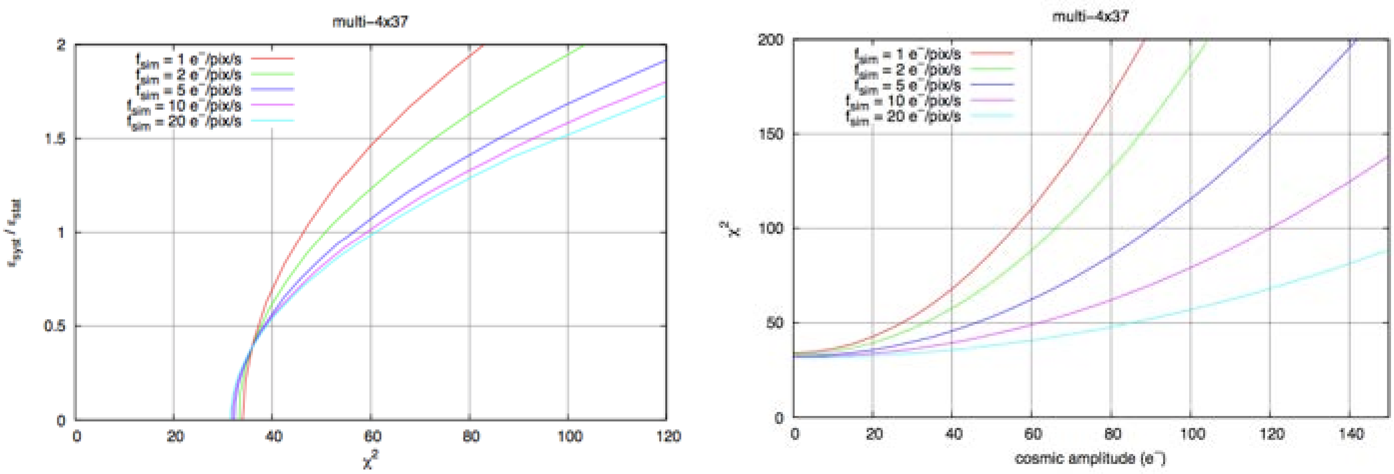

***Figure 9**: Illustration on the χ^2 flag setting. On the left, we define the threshold adapted to not introduce an error larger than the Poissonien one. On the right, we relate the cosmic amplitude detected with this threshold. Readout mode is multi-4x37*

- On left side, we plot the systematic error introduced by the cosmic in the fit over the statistical one: we should stay under 1: we see that all cases where the χ^2 is above 45 have a systematic error.
- On the right side, we have represented the χ^2 value in function of the cosmic amplitude for different signal flux: we see that the χ^2 threshold of 45 allows to detect all cosmic rays > 80 e- . Cosmic rays between 80 e- and 20 e- will be detected depending of the signal flux. A cosmic of 20 e- will be only seen for flux < 1 e/s/pixel.

This illustrates the power of the χ^2 value to take the 'statistical fluctuation into account' and then to detect different cosmic amplitudes.

The χ^2, when calculated on board and send on ground, can be used as a flag to decide if there is or not an anomaly in the pixel as describe upper.

If there is some freedom on board, we can also set an 'on board' threshold on the χ^2 to decide to transfer all points of the ramp of any pathologic pixels above this threshold:

- Without any telemetry limitation, it is possible to send the ramp information of a pixel to ground as soon as the χ^2 is too large, based on previous method, and then cosmic can be corrected on ground from the ramp.
- If the telemetry limits the number of ramp that can be transferred on ground (in the limit of 2 to 5%), it is possible to define a χ^2 threshold to transfer 'the worst' pixel ramps. The value of the χ^2 threshold in this case can be directly derived from the χ^2 probability in figure 3, where the criteria is the rate of pixels which are below some χ^2 probability value.

Let also emphasize that if the χ^2 is working well on board, the χ^2 value itself can be used not only to identify the anomaly but also to define a correction on ground. Cosmic rays, as transient events, are easy to distinguish from hardware anomalies that will be there for more than one exposure on a pixel. Then, from Figure 9, the χ^2 value of a 'transient pixel', gives directly the level of the cosmic amplitude to be removed if we have an estimation of the flux.

This needs to be studied in more details to estimate the efficiency of such a correction, but it will be a promising way to add a correction on the detection on ground. This implies to have the χ^2 value on ground.

7 Readout mode optimization

If we propose to transfer all points of a ramp of pathologic pixels (in the limit of 2 to 5%), the limitation is also the number of groups as the transfer of the full ramp means to keep all information in memory. To do so, the possibility to reduce the number of groups per exposure has been studied.

In spectroscopy, we assume that 150 frames are needed to reduce the noise. The way the 150 frames are distributed in an exposure to build groups has a low influence on the total noise. However, reducing the total number of read strongly impacts the total noise of the exposure.

Furthermore, there should exist a compromise between the number of frames per group and the number of groups per exposure which allow the best anomalies rejection. This section aims to define it. To do so, we define the recovery ratio as the proportion of superposition between the χ^2 distributions of the 'normal' pixels and the one obtained with a given anomaly. If the recovery ratio is null, 100% of the considered anomaly can be identified. In the multi-4x37 readout mode, it is for example the case of any anomaly which presents a χ^2 value higher than 45, as shown on figure 3 and 9.

We have looked at the overlap of the χ^2 distribution in a normal case and when we add an anomaly as a cosmic ray. We plot for example the 'rate of recovery' which is the overlap between the χ^2 distribution with and without the anomaly for a cosmic of 80 e- and different mode (m,n). Lower is the recovery rate, better is the discrimination.

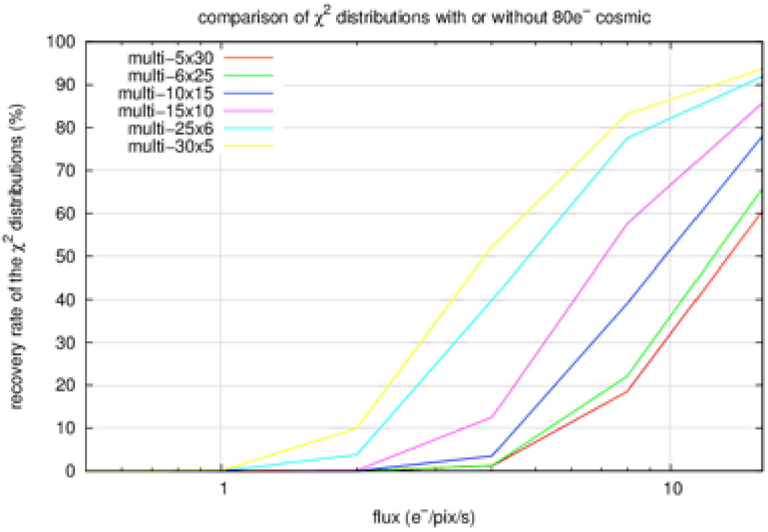

Figure 10: *Recovery rate of χ^2 distributions with or without a cosmic ray of 80 e^- as a function of the flux Total number of frames is fixed to 150*

As shown on this figure, more we have groups, better it is to detect a cosmic. Also, the lowest is the flux, the most powerful is the detection of anomalies. We can estimate then, the mode with the minimum of groups to reach a good cosmic rejection in the science range: any readout mode with a number of groups per exposure larger than 15 should allow to detect anomalies which amplitude is higher than 80 e^- and a flux lower than 5 e-/s/pixel. For higher flux, 10 to 50 % of cosmic will be detected depending of the mode and of the threshold.

On the basis of these simulations and with a view to reduce the number of groups per exposure to allow their transfer, a multi-accum readout mode with > 15 groups seems to be the best compromise. Figure 11 shows the result for the threshold for a multi 10x15 mode for which the mean χ^2 is 11.

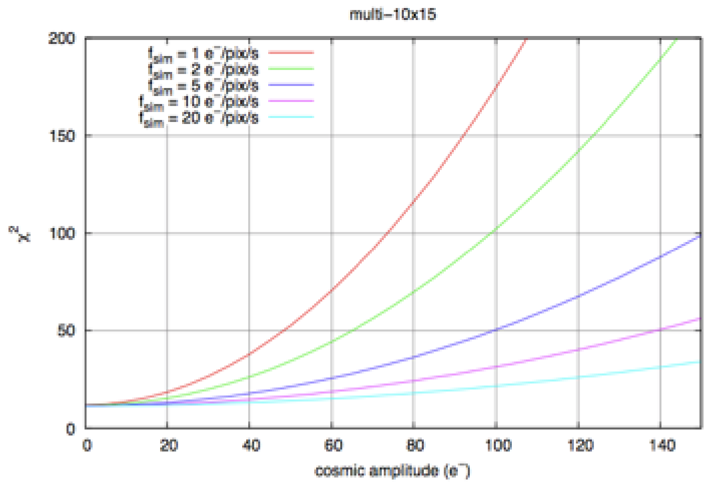

Figure 11*: Illustration on the χ^2 threshold setting: we define the threshold adapted to the mode and cosmic amplitudes. Readout mode is multi-10x15*

8 Recommendation to support changes

As shown in this document, the fit of the ramp can be done with a weighted least square fit on board using the error of co-added points and the difference between the groups. This way allows the error of the fit value to be Poisson distributed and then is a better approach, in the flux science range, than when computed with a regression without errors.

In all cases, detecting pathologic cases or cosmic rays, needs to dispose of an extra information.

We recommend to use a χ^2 quality test to do so. We emphasize it can be computed from the weighted least square fit, including the readout value, to be correctly evaluated. We see that the estimator is robust, can be easily computed on board and is very sensitive to any anomalies

We propose then to send the χ^2 value on ground as this will give us an information on ground of the level of reliability of the fit.

We propose also to transfer all points of the most pathologic pixels (i.e. very high χ^2, not previously masked). Using the probability of the χ^2, the threshold on the χ^2 value can be fixed to allow to transfer the ramp of not more than 2 to 5% of pixels.

Considering this study, we recommend also to reduce the number of groups per exposure in spectroscopy to a number, which reduces computation, but allow cosmic detection. We find that 15 groups is a good compromise between telemetry limitation and cosmic rejection criteria.